THE FAILURE OF THICK-WALLED CYLINDERS UNDER INTERNAL PRESSURE

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[*Manuscript received August* 17, 1956]

Summary

A number of thick-walled hollow cylinders of Nickel-Chrome-Molybdenum steel have been subjected to internal pressures up to 12,000 atm at room temperature and their yield points determined. The results conform to the Leinss (1955) empirical relation between yield pressure and tensile yield stress.

I. Introduction

The design of high-pressure equipment almost invariably requires a knowledge of the maximum internal pressure a cylindrical vessel of given material and known wall thickness will support. The well-known theory of Lamé (1852) derives the stresses and strains in the walls of a thick cylinder subjected to internal pressure, assuming the material to be elastic. From this theory and various criteria for failure of the material, several different relations can be derived between the internal pressure causing failure and the ratio K of outer to inner diameters of the cylinder. Even the most optimistic of these leads to the conclusion that an infinitely thick cylinder cannot support an internal pressure greater than the ultimate tensile stress for the material.

It has long been known, however, from the work of Bridgman (1914*a*) and others, that steel cylinders for which K is about 9 will support pressures up to 20,000 atm, well above the ultimate tensile stress for the steel. The layers of metal near the bore of such cylinders are highly stressed and become plastic but the cylinder does not fail until the plastic-elastic boundary has moved some distance from the bore.

The existence of overstrained metal near the bore is utilized in the autofrettage process for giving greater strength to gun-barrels; the theory and practice of this process have been described by Macrae (1930) and others. Manning (1945) points out that this analysis is restricted to the situation in which the degree of overstrain is small, and has developed a theory based on stress-strain relations obtained from torsion tests in which the degree of overstrain is large.

Recently, Crossland and Bones (1955*a*, 1955*b*) have confirmed Manning's theory by carrying out bursting tests to 6000 atm on mild steel cylinders. Many consider, however, that computation of bursting pressures from Manning's formulae is laborious and complex and there is a strong desire to relate bursting pressure to the results of tensile rather than torsion tests. Leinss (1955) pointed out that the results of Crossland and Bones can be represented by the relation

$$\frac{(K-1)\sigma_u}{P_b} = \alpha = \alpha_1+\beta(K-1), \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

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where K is the ratio of outer to inner diameters of a cylinder,

P_b is the bursting pressure of the cylinder,

σ_u is the ultimate tensile stress of the material,

a_1 and β are parameters independent of K.

Leinss showed that, since $(K-1)\sigma_u$ is the bursting pressure of a thin cylinder a_1 should be unity.

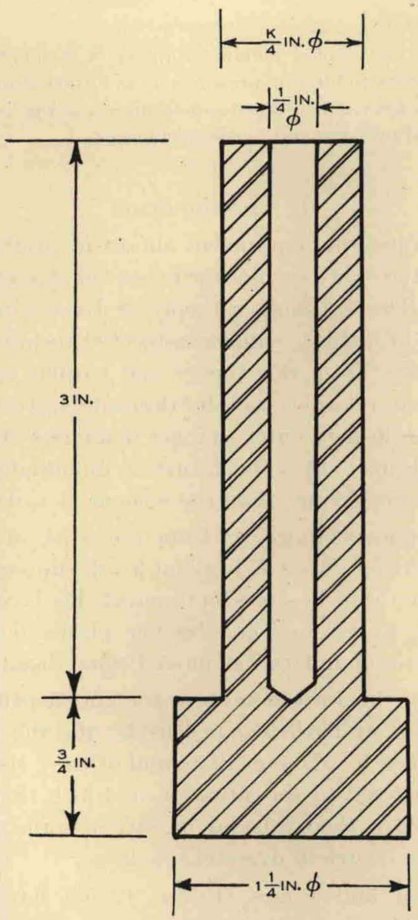

Fig. 1.—Test cylinder, longitudinal section.

The Leinss equation has the virtue of simplicity and provides the desired relationship between bursting pressure and tensile strength. Analysis of some published data shows that in all cases results can be represented by equations of the form (1) and it was decided to seek further experimental verification. At the same time it appeared desirable to investigate the variation of β with the tensile strength of the material.

II. Experimental

(a) *Materials*

A Nickel-Chrome-Molybdenum steel,* often used in this Laboratory for high-pressure cylinders, was chosen as a material of which the tensile strength could be varied by heat-treatment. It was used in three conditions: (i) fully annealed by prolonged heating at 770°C, (ii) tempered at 600°C after hardening, and (iii) tempered at 500°C after hardening. Hardening was effected by quenching in oil from 830°C.

For each series of experiments a set of cylinders, all of $\frac{1}{4}$ in. inner diameter but with various outer diameters, was used. Other dimensions of the cylinders are shown in Figure 1. In conditions (i) and (ii) of the steel most of the machining was done after heat-treatment, but in condition (iii) the cylinders were machined to correct size from annealed stock, heat-treated in salt baths, and then required only slight lapping of the bore before use. With each series a tensile test bar was made to British Standard No. 18 (with screwed-ends) and underwent exactly the same heat-treatment as the cylinders.

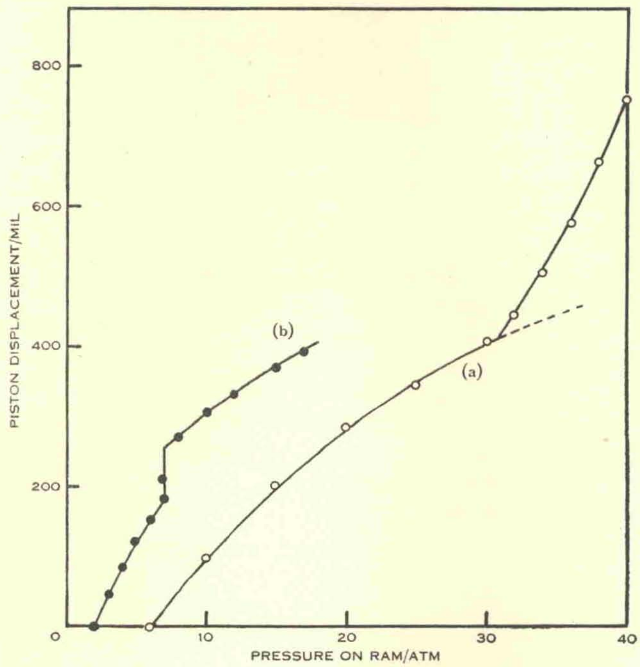

Fig. 2.—Typical piston displacement-pressure curves.
(*a*) Yielding of cylinder. (*b*) Phase transition.

(b) *Procedure*

The test bar was subjected to tension in a Greenwood and Batley 100,000-lb hydraulic testing machine, using a Gerard extensometer with dial gauge calibrated in $\frac{1}{10}$ mil. Extensions on a 2-in. gauge length were read to $\frac{1}{100}$ mil by visual inter-

* S.D.50 steel, manufactured by the Eagle and Globe Co. Makers' analysis: C, 0·30 – 0·37; Ni, 2·5–2·8; Cr, 0·6–0·8; Mo, 0·4–0·6; Mn, 0·5–0·7 per cent.

polation on the gauge. The observed extension was plotted against applied load and the stress value at which the plot ceased to be linear was taken as the yield point of

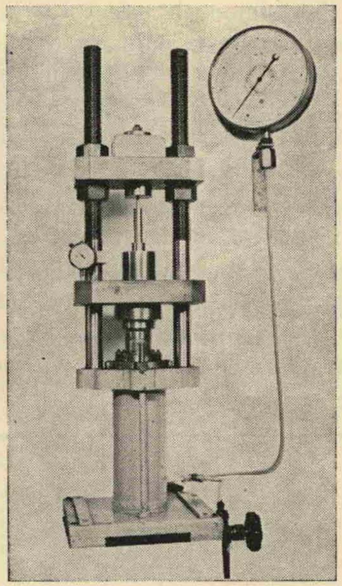

Fig. 3.—The press with test cylinder in position.

the specimen. The load at rupture was observed and the ultimate stress calculated on the basis of the original cross section of the test bar.

TABLE 1

RESULTS OF TENSILE TESTS

Series No.	Heat-Treatment (°C)	Yield Stress (tons/in^2)	Ultimate Stress (tons/in^2)	Reduction of Area (%)	Elongation (% on 2 in.)	Hardness (Rockwell c Scale)
1	Fully annealed 770	28·9	55	58	26	26
2	Quench 830 Temper 600	61·2	73·5	62	19·5	31
3	Quench 830 Temper 500	71·4	80·4	56	18·5	38

In testing a cylinder, it was filled with petroleum ether* and pressure in it developed by the advance of a piston actuated by a 25-ton hydraulic press. The

* This was used since it is known not to freeze at room temperature under pressures up to 20,000 atm.

fluid in the cylinder was retained by means of an unsupported area seal as described by Bridgman (1914*a*). The displacement of the piston was observed by means of a dial gauge calibrated in mils. For successive small equal increments of pressure on the ram of the press the increments of dial gauge reading were noted; these decreased as the compressibility of the fluid decreased until a sharp increase denoted yielding at the bore of the test cylinder. This is illustrated in curve (*a*) of Figure 2. The pressure on the hydrualic ram at this point was used to calculate the yield pressure of the test cylinder.

TABLE 2

RESULTS OF PRESSURE TESTS

Series No.	Cylinder No.	K	Yield Pressure (atm)	$(K-1)\sigma_y$ (atm)	a	a_1	β
1	1	1·50	1950	2200	1·13	1·00	0·267
	2	2·00	3460	4400	1·27		
	3	3·00	5710	8800	1·54		
	4	3·50	6470	11000	1·70		
	5	4·00	7400	13200	1·78		
2	1	1·50	3740	4665	1·25	1·01	0·477
	2	2·00	6280	9330	1·49		
	3	2·50	8350	13995	1·68		
	4	3·00	9470	18660	1·97		
	5	3·50	10230	23325	2·28		
	6	4·00	11730	27990	2·39		
	6A*	1·60	4320	5600	1·30		
3	1	1·51	4020	5550	1·38	1·0†	0·616
	2	1·77	5530	8380	1·52		
	3	2·02	7120	11100	1·56		

* After testing, cylinder No. 6 (1 in. outer diam.) was bored out to $\frac{5}{8}$ in. inner diam., thus removing metal which had suffered permanent deformation. It was then retested in a larger press giving the yield pressure shown.

† With fewer results and rather more scatter than in the other series the correlation was assumed to pass through the point = 1, $K = 1$.

After a test the outer diameter of a cylinder was found to have increased by from 1 to 3 mils, indicating that permanent set had taken place.

It will be seen that, under the conditions of test no axial stress occurred in the cylinder walls due to the pressure, so that the cylinders were effectively open-ended. Earlier experimental work of this nature has mostly been made on cylinders with closed-ends.

The correlation between ram pressure and the pressure developed in the test cylinder was found by observing pressure-induced phase transitions in several liquids. This was done in a hardened steel cylinder of 1 in. outer diameter and the same bore as the test cylinders. Such phase transitions are accompanied by a volume decrease at constant pressure (illustrated in curve (*b*) of Fig. 2) and this was detected on the dial gauge mounted as before. The pressure in the cylinder was then assumed to be the transition pressure given by Bridgman. The liquids used were bromobenzene (Bridgman 1915); chloroform, carbon tetrachloride (Bridgman 1914*b*); and water (Bridgman 1911). A linear equation between developed pressure and ram pressure was fitted to these results by the method of Least Squares.

The press with test cylinder in position is presented in Figure 3.

TABLE 3

PUBLISHED DATA

No.;* Reference	Material	Type of Failure	Relevant Tensile Stress (ton/in.^{2})	No. of Tests	Range of K	a_1	β	S.D. in a
1 Cook and Robertson (1911)	Mild steel	Yielding	15·2–16·5	27	1·35–3·65	0·820	1·362	0·08
2 Macrae (1930)	Nickel steel	,,	32·3–43·2	4	2·15–2·50	0·506	2·043	0·03
3 Cook (1932)	Mild steel	,,	19·5	9	1·40–3·00	0·672	1·466	0·03
4 Cook (1932)	,,	,,	19·5	7	2·70–3·94	0·637	1·376	0·01
5 Cook (1932)	,,	,,	19·5	6	3·00–7·00	0·466	1·304	0·04
6 Cook (1934)	,,	,,	23.4	6	1·17–4·00	0·866	1·559	0·04
7 Cook and Robertson (1911)	Cast iron	Bursting	8·3–12·0	8	1·30–2·96	0·867	0·770	0·08
8 Cook and Robertson (1911)	Mild steel	,,	24·3–26·3	9	1·35–1·79	0·954	0·584	0·06
9 Crossland and Bones (1955)†	,,	,,	32·9	9	1·33–3·72	1·023	0·467	0·02

* Numbers identify the sets of results on Figures 4 and 5.

† This analysis taken from Leinss (1955).

III. RESULTS

The Leinss factor a was calculated for each cylinder from the relation

$$a = \frac{(K-1)\sigma_y}{P_y}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where σ_y is the yield stress found from the tensile test,

P_y is the yield pressure of the cylinder.

For each series of cylinders the values of a were correlated according to equations of the form (1) and the parameters a_1 and β determined. Table 1 shows the results of the tensile tests and Table 2 the results of the pressure tests.

The analysis of previously published data is summarized in Table 3, individual plots of a against K being shown in Figures 4 and 5.

IV. Discussion

There is little doubt that the Leinss (1955) equation,

$$\frac{(K-1)\sigma}{P} = a_1+\beta(K-1), \qquad \text{.................(3)}$$

can be used to define the failure pressure of a thick cylinder by bursting or yielding by putting σ equal to the ultimate tensile stress σ_u or the tensile yield stress σ_y as applicable. The parameter a_1 is less than unity for most results previously published.

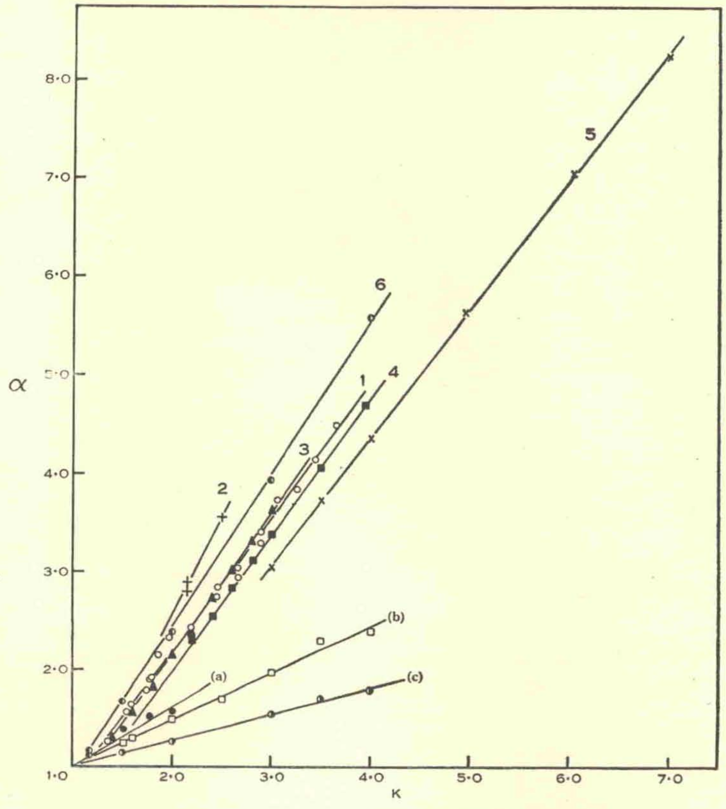

Fig. 4.—a,K plots for cylinders failing by yielding. Numbers on curves identify results in Table 3; (*a*), (*b*), (*c*) are results of present investigation.

The result for cylinder No. 6A of series 2 (Table 2) is of interest. This cylinder had a bore diameter different from that of the others in the series but fitted the correlation between them well. This indicates, as do the results of Crossland and Bones for bore diameters ranging from 0·269 to 0·750 in., the absence of the size effect reported by Cook (1932).

The experimental results obtained in this Laboratory indicate that, for a given material, the slope β of the a,K-line varies smoothly with the yield stress σ_y. The three pairs of values of β and σ_y were fitted to the equation,

$$\beta = A\sigma_y{}^a, \qquad \text{...................(4)}$$

giving $A = 1{\cdot}39 \times 10^{-2}$ and $a = 0{\cdot}876$. The maximum difference between experimental and calculated values of β was 6·4 per cent.

Further confirmation of some such relation would enable predictions of yield pressure for cylinders of a particular material to be made from the results of tensile tests alone.

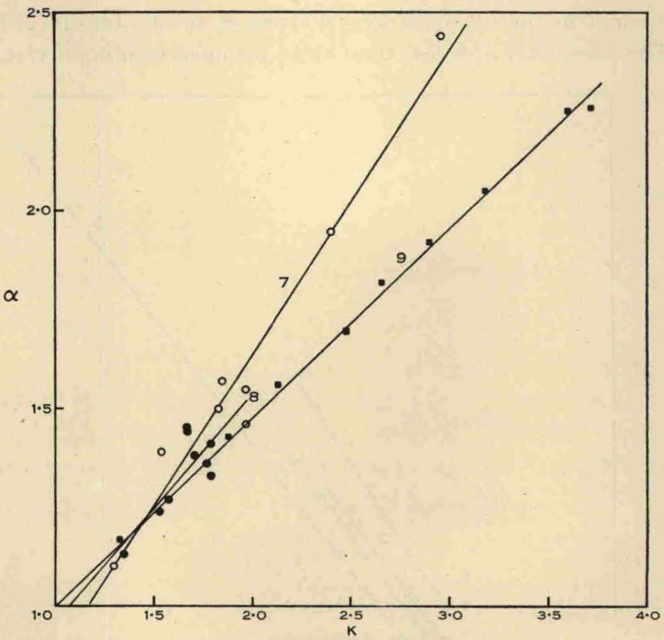

Fig. 5.—α,K plots for cylinders failing by bursting. Numbers on curves identify results in Table 3.

It is perhaps of interest to examine the implications of the Leinss equation (3). It may be transformed to

$$\frac{P}{\sigma} = \frac{1}{\alpha_1/(K-1)+\beta}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

As $K \to \infty$, $\alpha_1/(K-1) \to 0$, and in the limit $P_\infty = \sigma/\beta$, where P_∞ is the failure pressure for an infinitely thick cylinder. If, therefore, $\beta < 1$, $P_\infty > \sigma$, that is, for materials for which β is fractional, the failure pressure for thick cylinders of the material is greater than the relevant tensile stress.

P_∞ can be logically expected to increase with σ and this may be expressed

$$\frac{dP_\infty}{d\sigma} > 0. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

For the form of equation between β and σ chosen (eqn. (4)) the only condition for this is $a < 1$. The experimental results satisfy this condition.

No theoretical justification for the Leinss equation can be offered at the moment. The theoretical analysis of Turner (1910) gives, for a cylinder in which the plastic-elastic boundary has reached the outer surface

$$\frac{P_y}{\sigma_y} = \log_e K, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

a result obtained by van Iterson (1912) by a slightly different method. Using this the Leinss factor a may be derived

$$= \frac{(K-1)\sigma_y}{P_y} = \frac{K-1}{\log_e K}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

For values of K between 1·5 and 4·0 this approximates to a linear relation between a and K with a slope of 0·4. It allows, however, no variation of slope with σ_y and, therefore, cannot fully describe the experimental results.

The Leinss equation remains a most useful empirical relation between easily measured properties of a metal and the maximum allowable internal pressure for a cylinder of that metal.

V. Acknowledgments

The author is indebted to Mr. H. J. Vogan, Department of Civil Engineering, University of Sydney, for carrying out the tensile tests; to Mr. S. J. Lake for fabrication of test cylinders and bars; and to Dr. S. D. Hamann, High Pressure Laboratory, C.S.I.R.O., University of Sydney, for valuable discussions and advice.

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plastic-elastic boundary [illegible] the outer radius

$$\log K \quad [illegible] \qquad (7)$$

a result obtained by van Bueren (1947) by a slightly different method. Using this the Lamé factor α may be derived:

$$[illegible] \qquad (8)$$

For values of K between 1.5 and [illegible] approximates to a linear relation between α and K with a slope of [illegible]. [illegible] however no variation of slope with [illegible] fully the [illegible] experimental results.

The Lamé equation remains a most useful empirical relation between easily measured properties of a metal and the maximum allowable internal pressure for a cylinder of that metal.

V. Acknowledgments

The author is indebted to Mr. [illegible], Department of Civil Engineering, University of Sydney, for carrying out the tensile tests; to Mr. [illegible] for fabrication of test cylinders and bolts; and to Dr. [illegible], High Pressure Laboratory, [illegible] University of Sydney, for valuable discussions and advice.

VI. References

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